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g-BINARY δ-SEMI-CONTINUOUS FUNCTIONS IN g-BINARY TOPOLOGICAL SPACES

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ABSTRACT

In this paper we introduce and study the concept g-binary δ-semi-continuity in g-binary topological spaces and investigate various relationships.

Keywords: *g-binary open sets, g-binary semi-open sets, g-binary pre-open sets, g-binary δ-semi-continuous, totally g-binary δ-semi-continuous, strongly g-binary δ-semi-continuous functions.*

I. INTRODUCTION

Levine [8] introduced semi open and semi continuous functions in topological spaces. Recently the authors [15] introduced the concept of binary topology between two sets and investigate some of the basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. In this paper we introduce and study g-binary δ-semi-continuity in g-binary topological spaces and investigate various relationships. Section 2 deals with the basic concepts of g-binary topological spaces. In section 3 g-binary δ-semi-continuity in g-binary topological spaces are studied and established the relationships. Throughout the paper $\wp(x)$ denotes the power set of x.

II. PRELIMINARIES

Definition 2.1: Let X and Y are any two non-empty sets. A g-binary topology from X to Y is a binary structure $M_g \subseteq \wp(X) \times \wp(Y)$ that satisfies the following axioms:
$(\emptyset, \emptyset)$ and $(X, X) \in M_g$

If $\{(A_\alpha, B_\alpha) ; \alpha \in \Delta\}$ is a family of members of M_g, then $(\bigcup_{\alpha\in\Delta} A_\alpha , \bigcup_{\alpha\in\Delta} B_\alpha) \in M_g$
If M_g is a g-binary topology from X to Y, then the triplet (X, Y, M_g) is called a g-binary topological space and the members of M_g are called the g-binary open subsets of the g-binary topological space (X, Y, M_g). The elements of $X \times Y$ are called the g-binary points (or g-binary sets) of g-binary topological space (X, Y, M_g).

Definition 2.2: Let (X, Y, M_g) be a g-binary topological space and $A \subseteq X, B \subseteq Y$. Then (A, B) is g-binary closed in (X, Y, M_g) if $(X\backslash A, Y\backslash B) \in M_g$.

Definition 2.3: In a g-binary topological space (X, Y, M_g) if $(A, B) \subseteq (X, Y)$, then $gbcl(A, B)$ is smallest g-binary closed set containing (A, B).

Proposition 2.1: Let $(A, B) \subseteq (X, Y)$. Then (A, B) is g-binary closed in (X, Y, M_g) iff $(A, B) = gbcl(A, B)$.

Definition 2.4: In a g-binary topological space (X, Y, M_g) if $(A, B) \subseteq (X, Y)$, then $gbint(A, B)$ is largest g-binary open set contained in (A, B).

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Proposition 2.2: Let $(A, B) \subseteq (X, Y)$. Then (A, B) is g-binary open in (X, Y, M_g) iff $(A, B) = gbint(A, B)$.

Definition 2.5: A subset (A, B) of a g-binary topological space (X, Y, M_g) is called
g-binary semi-open if $(A, B) \subseteq gbcl(gbint(A, B))$.
g-binary pre-open if $(A, B) \subseteq gbint(gbcl(A, B))$.

Definition 2.6: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is said to be
g-binary continuous if $f^{-1}(A, B)$ is g-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).
g-binary semi-continuous if $f^{-1}(A, B)$ is g-semi-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).
g-binary pre-continuous if $f^{-1}(A, B)$ is g-pre-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

Definition 2.7: Let (X, Y, M_g) be an g-binary topological space and (A, B) be a subset of $\wp(X) \times \wp(Y)$, then $gbcl_\delta(A, B) = \{(x, y) \in \wp(X) \times \wp(Y) : gbint\big(gbcl(U, V)\big) \cap (A, B) \neq \emptyset, (U, V) \in M_g \text{ and } (x, y) \in (U, V)\}$

III. G-BINARY δ-SEMI-CONTINUOUS FUNCTIONS

Definition 3.1: A subset (A, B) of a g-binary topological space (X, Y, M_g) is called g-binary **δ**-semi-open set if $(A, B) \subseteq gbcl(gbint_\delta(A, B))$.

Definition 3.2: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is said to be g-binary **δ**-semi-continuous if $f^{-1}(A, B)$ is g-**δ**-semi-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

Example 3.1: Let $Z = \{1, 2, 3\}$, $X = \{a_1, a_2\}$ and $Y = \{b_1, b_2\}$.Then $\tau = \{\emptyset, \{1,2\}, \{2,3\}, Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_2\}, \{b_2\}), (\{a_1\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_2, b_2) = f(3)$ and $f(2) = (a_1, b_1)$. Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_2\}, \{b_2\}) = \{1,3\}$, $f^{-1}(\{a_1\}, \{Y\}) = \{2\}$ and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open set in (X, Y, M_g) is g-**δ**-semi-open in (Z, τ). Hence f is g-binary δ-semi-continuous.

Remark 3.1: The concepts of g-binary continuity and g-binary **δ**-semi-continuity in g-binary topology are independent as shown in Example 3.2 and Example 3.3.

Example 3.2: Let $Z = \{1, 2, 3\}$, $X = \{a_1, a_2\}$ and $Y = \{b_1, b_2\}$.Then $\tau = \{\emptyset, \{1\}, \{1,2\}, \{2,3\}, Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_1, b_1)$ and $f(2) = f(3) = (a_2, b_2)$ Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_1\}, \{b_1\}) = \{1\}$, $f^{-1}(\{a_2\}, \{Y\}) = \{2,3\}$ and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open set in (X, Y, M_g) is g-open in (Z, τ). Hence f is g-binary continuous but not g-binary δ-semi-continuous because the set $\{1\}$ is g-open in (Z, τ) but not g-δ-semi-open.

Example 3.3: In Example 3.1 f is g-binary δ-semi-continuous but not g-binary continuous because the set $\{1,3\}$ is g-δ-semi-open in (Z, τ) but not g-open.

Definition 3.3: Let (Z, τ) be a g-topological space and (X, Y, M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is called totally g-binary continuous if $f^{-1}(A, B)$ is g-clopen in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

Definition 3.4: Let (Z,τ) be a g-topological space and (X,Y,M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is called totally g-binary δ-semi-continuous if $f^{-1}(A,B)$ is g-δ-semi-clopen in (Z,τ) for every g-binary open set (A, B) in (X,Y,M_g).

Example 3.4: Let $Z = \{1,2,3\}, X = \{a_1,a_2\}$ and $Y = \{b_1,b_2\}$. Then $\tau = \{\emptyset,\{1\},\{1,2\},\{2,3\},Z\}$ and $M_g = \{(\emptyset,\emptyset),(\{a_1\},\{b_1\}), (\{a_2\},\{Y\}),(X,Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y.

Define $f: Z \to X \times Y$ by $f(1) = (a_1,b_1) = f(2)$ and $f(3) = (a_2,b_2)$. Now $f^{-1}(\emptyset,\emptyset) = \emptyset$, $f^{-1}(\{a_1\},\{b_1\}) = \{1,2\}$, $f^{-1}(\{a_2\},\{Y\}) = \{3\}$ and $f^{-1}(X,Y) = Z$. This shows that the inverse image of every g-binary open set in (X,Y,M_g) is g-δ-semi-clopen in (Z,τ). Hence f is totally g-binary δ-semi-continuous.

Remark 3.2: The concepts of totally g-binary continuity and totally g-binary δ-semi-continuity in g-binary topology are independent as shown in Example 3.5 and Example 3.6.

Example 3.5: Let $Z = \{1,2,3\}, X = \{a_1,a_2\}$ and $Y = \{b_1,b_2\}$. Then $\tau = \{\emptyset,\{1\},\{1,2\},\{2,3\},Z\}$ and $M_g = \{(\emptyset,\emptyset),(\{a_1\},\{b_1\}),(\{a_2\},\{Y\}),(X,Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y.

Define $f: Z \to X \times Y$ by $f(1) = (a_1,b_1)$ and $f(2) = (a_2,b_2) = f(3)$. Now $f^{-1}(\emptyset,\emptyset) = \emptyset$, $f^{-1}(\{a_1\},\{b_1\}) = \{1\}$, $f^{-1}(\{a_1\},\{Y\}) = \{1\}$, $f^{-1}(\{a_2\},\{Y\}) = \{2,3\}$ and $f^{-1}(X,Y) = Z$. This shows that the inverse image of every g-binary open set in (X,Y,M_g) is g-clopen in (Z,τ). Hence f is totally g-binary continuous but not totally g-binary δ-semi-continuous because the set $\{2,3\}$ is g-clopen in (Z,τ) but not g-δ-semi-clopen.

Example 3.6: In Example 3.4 f is totally g-binary δ-semi-continuous but not totally g-binary continuous because the set $\{1,2\}$ is g-δ-semi-clopen in (Z,τ) but not g-clopen.

Definition 3.5: Let (Z,τ) be a g-topological space and (X,Y,M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is called strongly g-binary continuous if $f^{-1}(A,B)$ is g-clopen in (Z,τ) for every g-binary set (A, B) in (X,Y,M_g).

Definition 3.6: Let (Z,τ) be a g-topological space and (X,Y,M_g) be g-binary topological space. Then the function $f: Z \to X \times Y$ is called strongly g-binary δ-semi-continuous if $f^{-1}(A,B)$ is g-δ-semi-clopen in (Z,τ) for every g-binary set (A, B) in (X,Y,M_g).

Example 3.7: Let $Z = \{1,2,3\}, X = \{a_1,a_2\}$ and $Y = \{b_1,b_2\}$. Then $\tau = \{\emptyset,\{1\},\{1,2\},\{2,3\},Z\}$ and $M_g = \{(\emptyset,\emptyset), (\{a_1\},\{b_1\}),(\{a_2\},\{Y\}),(X,Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_1,b_1) = f(2)$ and $f(3) = (a_2,b_2)$. Now $f^{-1}(\emptyset,\emptyset) = \emptyset$, $f^{-1}(\{a_1\},\{b_1\}) = \{1,2\}$, $f^{-1}(\{a_1\},\{Y\}) = \{1,2\}$, $f^{-1}(\{a_2\},\{Y\}) = \{3\}$, $f^{-1}(\{\emptyset\},\{b_1\}) = \{\emptyset\}$, $f^{-1}(\{\emptyset\},\{b_2\}) = \{\emptyset\}, f^{-1}(\{\emptyset\}, \{Y\}) = \{\emptyset\}$, $f^{-1}(\{a_1\},\{\emptyset\}) = \{\emptyset\}$, $f^{-1}(\{a_1\},\{ b_2\}) = \{\emptyset\}$, $f^{-1}(\{a_2\},\emptyset) = \{\emptyset\}$, $f^{-1}(\{a_2\},\{b_1\}) = \{\emptyset\}$, $f^{-1}(\{a_2\},\{ b_2\}) = \{3\}$, $f^{-1}(\{X\},\{\emptyset\}) = \{\emptyset\}$, $f^{-1}(\{X\},\{b_1\}) = \{1,2\}$, $f^{-1}(\{X\},\{b_2\}) = \{3\}$ and $f^{-1}(X,Y) = Z$. This shows that the inverse image of every g-binary set in (X,Y,M_g) is g-δ-semi-clopen in (Z,τ). Hence f is strongly g-binary δ-semi-continuous.

Remark 3.3: The concepts of strongly g-binary continuity and strongly g-binary δ-semi-continuity in g-binary topology are independent as shown in Example 3.8 and Example 3.9.

Example 3.8: Let $Z = \{1,2,3\}, X = \{a_1,a_2\}$ and $Y = \{b_1,b_2\}$. Then $\tau = \{\emptyset,\{1\},\{1,2\},\{2,3\},Z\}$ and $M_g = \{(\emptyset,\emptyset), (\{a_1\},\{b_1\}),(\{a_2\},\{Y\}),(X,Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_1,b_2)$ and $f(2) = (a_2,b_2) = f(3)$. Now

$f^{-1}(\emptyset,\emptyset)=\emptyset$, $f^{-1}(\{a_1\},\{b_1\})=\{\emptyset\}$, $f^{-1}(\{a_1\},\{Y\})=\{1\}$, $f^{-1}(\{a_2\},\{Y\})=\{2,3\}$, $f^{-1}(\{\emptyset\},\{b_1\})=\{\emptyset\}$, $f^{-1}(\{\emptyset\},\{b_2\})=\{\emptyset\}$, $f^{-1}(\{\emptyset\},\{Y\})=\{\emptyset\}$, $f^{-1}(\{a_1\},\{\emptyset\})=\{\emptyset\}$, $f^{-1}(\{a_1\},\{b_2\})=\{1\}$, , $f^{-1}(\{a_2\},\emptyset)=\{\emptyset\}$, $f^{-1}(\{a_2\},\{b_1\})=\{\emptyset\}$, $f^{-1}(\{a_2\},\{b_2\})=\{2,3\}$, $f^{-1}(\{X\},\{\emptyset\})=\{\emptyset\}$, $f^{-1}(\{X\},\{b_1\})=\{1\}$, $f^{-1}(\{X\},\{b_2\})=\{2,3\}$ and $f^{-1}(X,Y)=Z$. This shows that the inverse image of every g-binary set in (X,Y,M_g) is g-clopen in (Z,τ). Hence f is strongly g-binary continuous but not strongly g-binary δ-semi-continuous because $f^{-1}(\{a_1\},\{b_2\})=\{1\}$, where $\{1\}$ is g-clopen in (Z,τ) but not g-δ-semi-clopen.

Example 3.9: In Example 3.7 f is strongly g-binary δ-semi-continuous but not strongly g-binary continuous because the set $\{1,2\}$ is g-δ-semi-clopen in (Z,τ) but not g-clopen.
From the above discussion we have the following result:

g-binary continuous ⇎ g-binary δ-semi-continuous
Totally g-binary continuous ⇎ totally g-binary δ-semi-continuous
Strongly g-binary continuous ⇎ strongly g-binary δ-semi-continuous

IV. CONCLUSION

The concept of g-binary δ-semi-continuity in g-binary topological spaces is introduced and studied. Further different relationships between these functions are investigated.

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